

# Longitude by the Method of Lunar Distance

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Mar 21, 2005

Finding longitude at sea without a chronometer by the method of lunar distance is the most difficult and complex aspect of celestial navigation. It is unlikely, in the 21<sup>st</sup> century, that you would ever have to rely on this method to find your way home. However, a celestial navigator who thoroughly understands “lunars” will also have a sound grasp of all aspects of celestial navigation, and will be comfortable with any celestial navigation problem encountered. And for the celestial enthusiast, there is no greater satisfaction than dispensing with the chronometer and successfully finding a longitude at sea through careful lunar observation.

This article provides a concise but complete description of the lunar distance method, suitable for anyone who has basic familiarity with the use of a sextant to find position by celestial navigation. The only tools required are a good quality metal sextant (the plastic ones are not adequate for lunars), the Nautical Almanac, and an inexpensive scientific calculator. The iterative approach to the calculations used here simplifies the observation requirements, and allows concentration on carefully determining the Lunar Distance.

The article is divided into separate sections, which cover each aspect of the lunar distance problem:

1. A Short History of Lunars
2. Measuring the Observed Lunar Distance (OLD)
3. Averaging Multiple Observations to Reduce Error
4. A Brief Trigonometric Interlude
5. The Spherical Triangle and Calculator Methods
6. Calculating the OLD for a Known Position and Time
7. Determining GMT and Longitude at Sea from Observation of the Lunar Distance (OLD)
8. Finding an Actual Longitude at Sea

A reader with some familiarity with lunar distance concepts and an understanding of the spherical trigonometry used in navigation can go directly to sections 6 and 7, which describe the iterative method for calculating lunars. Someone approaching lunars for the first time would definitely find the other sections useful.

## 1. A Short History of Lunars

The stars with their fixed declinations, or the sun with its slowly varying one, have enabled people to determine latitude since antiquity by altitude at meridian passage. To

find longitude on a steadily spinning world, however, requires a clock. To the navigator time is longitude, 360 degrees every 24 hours, a degree every four minutes, a minute every four seconds, steadily ticking off as the world turns. Even the best pendulum clocks could not keep time on a tossing ship, and early mariners could only estimate their longitude by dead reckoning. Columbus, although he made contributions to navigation, had a particularly poor concept of longitude. In four voyages to the New World the lands he reached were 150 degrees of longitude from his estimated position, Cipango, the island of Japan, more than 7000 miles distant and on the other side of a different ocean. Clearly the Age of Discovery would require a better method of estimating position, and the Problem of Longitude engaged astronomers, navigators, and clock makers (as well as assorted mystics and crackpots) for the next 250 years.

There are other ways to tell time, however, besides a mechanical timepiece. The sky provides a number of clocks for those who know how to read them. While the explorers opened up the oceans, the astronomers expanded the heavens, breaking out of the confining celestial spheres of the earth-centered Ptolemaic theory and into the vast expanse of the Copernican universe. In 1611 Galileo devised tables of his newly discovered moons of Jupiter, which could be used to determine time and thus the longitude of the observer. These tables worked in theory, but 17<sup>th</sup> century sea captains were never able use them.

The moon though, does provide a practical method of telling time at sea. The moon passes through its cycle of phases moving 360 degrees with respect to the sun in 28 days, or about 30 minutes of arc per hour. If you can measure the angular distance between the moon and the sun to within 0.5 minutes of arc, you can determine time to within one minute. One minute of time is 15 minutes of longitude, or plus or minus 12 miles at my latitude 38 North. Not GPS accuracy, but good enough to map the continents, or enable a ship to make a safe landfall.

Lunar sights became a standard method of determining longitude. Even after the invention of the marine chronometer in 1761, lunar navigation remained important. Captain James Cook had no chronometer aboard the Endeavor in 1768; all Cook's first survey of the South Pacific was implemented by careful lunar observation. Throughout the 1800's on many vessels a navigator who could work lunars was the only way to find longitude, and still the best insurance for a safe return to port. Working a lunar, however, requires "Clearing the Lunar Distance", an intense set of calculations involving "endless fussing with logarithms". Nathaniel Bowdich described a systematic approach to working lunars in the *American Practical Navigator* in 1802, which made this technique available to a century of American sea captains. Lunar tables were published in *Bowdich* until 1914.

Joshua Slocum left his chronometer behind on his 1895 solo circumnavigation, because he lacked the funds to have it repaired. Of all Slocum's feats of seamanship and navigation, perhaps the most famous is his precise landfall after navigating across the Pacific Ocean with nothing but a sextant and a tin clock, which he kept running by boiling periodically. Slocum used dead reckoning and a lunar longitude.

Lunar longitude has become pretty much of lost art since the time of Slocum, and celestial navigation itself has been relegated to a hobby for sailors, or for those who want to practice the sailor's arts. With an inexpensive calculator, however, and a little persistence, anyone who practices celestial navigation can try their hand at lunar longitude by following the suggestions below.

## **2. Measuring the Observed Lunar Distance (OLD)**

Observed Lunar Distance (OLD) is the angular distance in the sky between the adjacent edges of the sun and the moon; its measurement is the way we read the lunar clock. To measure the Observed Lunar Distance with a sextant, you bring the image of the sun toward the moon until the sun's inner edge just touches the inner edge of the moon. As with all sextant measurements, you need to "rock" the sextant back and forth a bit to make sure the images are exactly tangent. The sextant reading will give the angular distance between the closest edges of the sun and the moon. Anyone who has tried "bringing the sun down" to the horizon will appreciate how much more difficult it can be to bring the image of the sun across the sky until it touches the image of the moon. Pre-calculating the approximate lunar distance and setting the sextant to that value makes the procedure a whole lot easier. Record the sextant reading and watch time for each sight. Make a series of measurements so that they can be averaged (see below) to get a more accurate result.

This measurement must be done very carefully; an error of just 1' in the lunar distance (an error of one mile in ordinary navigation) can lead to a 30-mile position error from the lunar calculation. We really need to make this measurement to an accuracy of a few tenths of a minute, pushing the limits of what can be done with a sextant. I use a six-power telescope on my sextant to more accurately determine the point of tangency.

The sextant should be carefully adjusted before attempting lunar distance measurements. Consult the instruction manual for the sextant for adjusting the index mirror to be perpendicular to the frame. Side error must then be rigorously eliminated, and the index error carefully determined. The most straightforward way to evaluate both side and index error is to observe a star at night with the sextant set to read near zero. The star will be split into two images. Turning the micrometer drum or tangent screw back and forth should cause the two images to pass through each other. If they do not, there is *side error*, and it must be removed by adjusting the horizon mirror. Once the side error is removed, the two star images can be brought into coincidence, and the sextant reading at that point gives the index error. If the index error is modest, less than a few minutes of arc, it doesn't need to be removed.

Another way to check for side and index error is to observe the sun with the sextant again set near zero. (See "How to Measure Index Correction Using the Sun" on [www.starpath.com](http://www.starpath.com).) Make sure you have the proper sun shades in place over both the

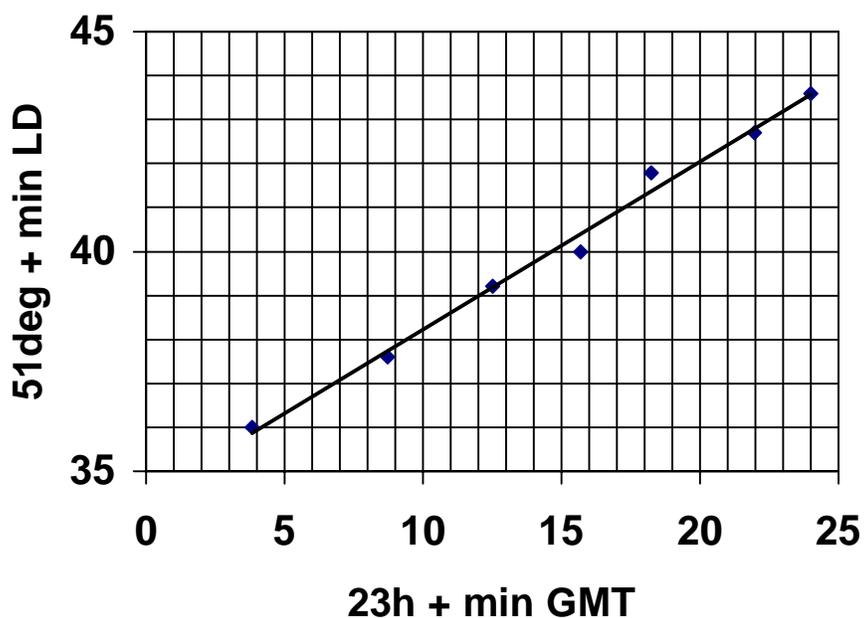
index and horizon mirrors before looking at the sun! Adjusting the micrometer drum should allow the two sun images to be brought into coincidence; otherwise, the horizon mirror must be adjusted to eliminate side error. Now adjust the micrometer drum to move the two sun images until they are just touching at one point of tangency. Record the sextant reading and turn the drum in the other direction until the two images are again just touching, and again record the reading. One of the readings will be a negative angle – “off the arc”. An “off the arc” reading of 28’ on the micrometer drum is really  $-1^{\circ} +28'$  or  $-32'$ . The difference between the two measurements is four times the sun semi-diameter as found in the Nautical Almanac, so you can check the accuracy of your results. One half the algebraic sum of the two values is the index error. It is good practice for the lunar distance measurement to repeat the sun index error determination a number of times, until you can do it accurately and consistently.

### 3. Averaging multiple observations to reduce error.

Here are some real lunar distance data taken by David Burch on May 7, 2000 from Lat  $47^{\circ} 40.5' N$ , Lon  $122^{\circ} 23.9' W$ , and published on the Starpath web site in 2004. Times are in GMT and Index error is 0.0’:

GMT	Observed Lunar Distance
23:03:49	$51^{\circ} 36.0'$
23:08:43	$51^{\circ} 37.6'$
23:12:30	$51^{\circ} 39.2'$
23:15:25	$51^{\circ} 40.0'$
23:18:15	$51^{\circ} 41.8'$
23:21:58	$51^{\circ} 42.7'$
23:24:00	$51^{\circ} 43.6'$

These observations can be averaged to reduce random error. Plot these data points on graph paper and draw the “best” straight line through them (see below). I also then threw out the worst point, and then calculated the best linear regression line through the remaining points using the linear regression function on my calculator. By whatever means a regression line is selected, that line then represents the averaging process. Any single point on the regression line represents the average of all the data, and there is no further need to deal with multiple observations in the analysis. See “How to Average Celestial Sights for Optimum Accuracy” by David Burch at [www.starpath.com](http://www.starpath.com).



I selected the observation at GMT 23:24:00, OLD  $51^{\circ}43.4'$  as the data average, as most of the reasonable regression lines go pretty much through that point. (I also cheated, and did a little side calculation unfairly using the known GMT and Longitude. I found that this observation was very good, only off by  $0.2'$  arc. This is better than the stated accuracy of my Astra sextant, and better than I can usually make real measurements.) We will use this observation to work the examples in the rest of the text.

#### 4. A Brief Trigonometric Interlude

If you know anything about trigonometry, you can skip this section. There are books written about trigonometry, but these few paragraphs will tell you everything you absolutely have to know to work a lunar by the methods below.

If you deal much at all with triangles either on a flat plane like a piece of paper, or on a sphere like the earth or the sky, you will confront the trigonometric functions, sine and cosine. The sine of an angle, written  $\sin(\text{angle})$ , is a number between  $-1$  and  $+1$  inclusively. You find the number by looking it up in a table, or better yet, punching the angle into a pocket calculator and hitting the “sin” key. The sine numbers are mostly infinite decimals, but we only need to pay attention to the first five figures. So for example:  $\sin(57^{\circ}) = .83867$ , approximately. The cosine of an angle, written  $\cos(\text{angle})$ , is (usually) a different number, likewise between  $-1$  and  $+1$  inclusively; you find it by punching the “cos” key on the calculator.  $\cos(57^{\circ}) = .54464$ , for example.

Just as you can go from the angle to the sine or cosine number, you can go from the sine or cosine number back to the angle. For every number between  $-1$  and  $+1$  inclusively,

there is an angle whose sine is that number. For example,  $57^\circ$  is the angle whose sine is .83867. We write that as:  $\sin^{-1} (.83867) = 57^\circ$ , where  $\sin^{-1}$  is called “the inverse sine”. You find the angle from the sine number by punching the number into a calculator and hitting the “ $\sin^{-1}$ ” key. The inverse cosine is similar.

## 5. The Spherical Triangle and Calculator Methods

All celestial navigation calculations involve solving spherical triangles using the formulas of spherical trigonometry. Those formulas require finding the sines and cosines of different of angles and multiplying and dividing a several 5 digit numbers together—a formidable task. All the sight reduction tables like HO-229 or HO-249 are tabulations of specialized solutions of the spherical triangle suitable for determining LOP’s from celestial sights. Lunar distance calculations involve solving a number of spherical triangles, and the calculations are even more extensive. Lunar longitude requires great precision both in measurements and calculations. None of the inspection tables you may be familiar with are suitable for lunar longitude calculations; such tables are neither accurate nor flexible enough. Those tables generally require integral degree values for Local Hour Angle and latitude, or some other argument, which doesn’t work for the calculations required here. Special tables for lunar distance have been published (Bruce Stark, *Tables for Clearing the Lunar Distance* available from StarPath), which are specialized log – trig tables. If you want to be a purist, you could use the standard log – trig tables included in Bowdich; they would work with the method described below, but you would have to want the answer very bad to go through all that effort.

However, an inexpensive scientific calculator will make short work of the messy formulas of spherical trigonometry. Make sure you get one with a “degrees – minutes-seconds” key. Such a key is invaluable both for entering angles and for working with the hours-minutes-seconds of time. I use a Casio fx-300MS, which I bought at Staples for less than \$10. You can key the problem into that calculator the same way it is written in the formula, which makes the calculations easy to do. A big advantage of calculator methods for all celestial navigation is that you can use your best Estimated Position (Dead Reckoning position) for your Assumed Position (AP) in the calculations. Then the Intercept is the distance the LOP is from your EP, and plotting is very much simplified. If you use a GPS position for your AP for practice, then the intercept is just your sextant measurement error, a good way to get fast, clear feedback on your technique.

FIGURE 1

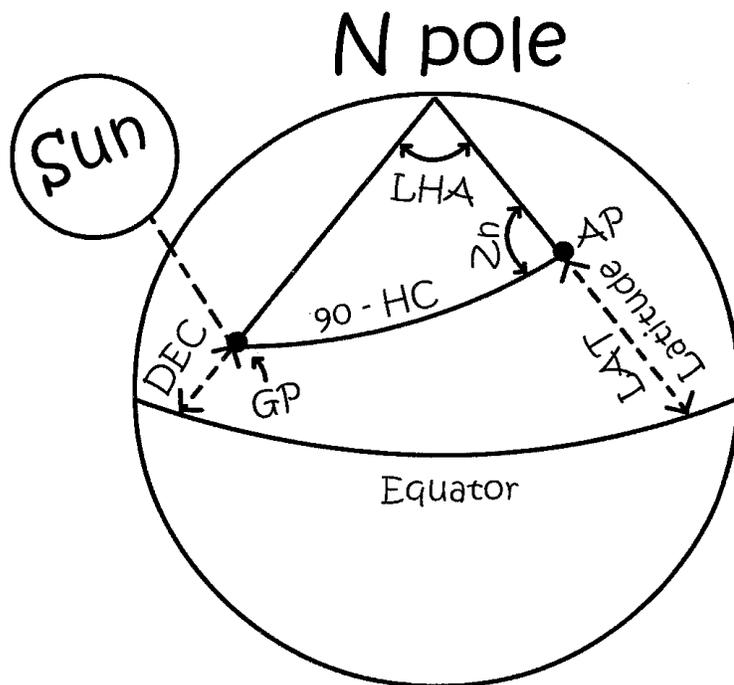


Figure one is the familiar Nautical Triangle showing the latitude (Lat) of the observer at the assumed position (AP), the declination (Dec) of the ground position (GP) of the celestial body, and the Local Hour Angle (LHA) between the GP and the observer. The  $LHA = GHA - Longitude$  (West longitude considered positive, East negative, in this convention). Then the calculated altitude (HC) and meridian angle (Z) of the body is given by the standard formulas of spherical trigonometry:

$$1) \sin(HC) = \sin(Dec)\sin(Lat) + \cos(Dec)\cos(Lat)\cos(LHA)$$

$$2) \cos(Z) = \frac{\sin(Dec) - \sin(HC)\sin(Lat)}{\cos(HC)\cos(Lat)}$$

Example: Find the altitude and Azimuth of the sun from Lat  $47^{\circ} 40.5' N$ , Lon  $125^{\circ} 53.9' W$  on 5/7/00 at 23:38:00 GMT.

Solution: In our example, the Nautical Almanac gives the position of the sun on 5/7/00 at 23:38:00 GMT as:

GHA  $175^{\circ} 22.8'$

Dec  $17^{\circ} 6.4'$

From our assumed position at Latitude  $47^{\circ} 40.5'$ , Longitude  $125^{\circ} 53.9'$  we get:

$$\text{LHA} = 175^\circ 22.8' - 125^\circ 53.9' = 49^\circ 28.9'$$

$$\sin(\text{HC}) = \sin(17^\circ 6.4')\sin(47^\circ 40.5') + \cos(17^\circ 6.4')\cos(47^\circ 40.5')\cos(49^\circ 28.9') = .63561$$

$$\text{HC} = \sin^{-1}(.63561) = 39^\circ 27.9'$$

$$\cos(Z) = \frac{\sin(17^\circ 6.4') - \sin(39^\circ 27.9')\sin(47^\circ 40.5')}{\cos(39^\circ 27.9')\cos(47^\circ 40.5')} = -.33813$$

$$Z = \cos^{-1}(-.33813) = 109.763^\circ$$

$$\text{Since the sun is to the west, the azimuth } \text{AZ} = 360^\circ - Z = 250.237^\circ = 250^\circ 14.2'$$

These same formulas can be applied to an analogous spherical triangle on the celestial sphere to find the angular distance between two celestial bodies, like between the sun and moon, for example. Celestial north and the celestial equator replace the north pole and the earth's equator, and the declinations of the two bodies replace the latitude and declination in figure 1. The "LHA" is the difference in GHA of the two bodies. Sin(HC) in equation 1 is replaced by Cos(Distance), where "Distance" is the angular separation between the two bodies. Similarly, the visible sky forms another sphere with the visible horizon analogous to the equator and the zenith corresponding to the pole. We will use triangles on this sphere to clear the lunar distance (see below).

## 6. Calculating the OLD for a Known Position and Time

Calculating the Observed Lunar Distance for a known position and time is the key step in finding longitude by lunar distance in the method described here. This procedure is basically the inverse of the possibly more familiar "clearing the lunar distance" frequently utilized in other descriptions of lunar longitude techniques.

The angle between the centers of the sun and moon, the lunar distance (LD), can be calculated for a given date and time from the declinations and GHA's of the sun and moon given in the Nautical Almanac, using the formulas of spherical trigonometry applied to the positions of the bodies on the celestial sphere. Unfortunately, this straightforward calculation gives the geocentric lunar distance, the angle that would be measured by an observer at the center of the earth, a particularly inconvenient vantage point for astronomical observations. For observations from the surface of the earth, the geocentric positions of the sun and moon are altered by the effects of parallax and atmospheric refraction to give the actual Observed Positions in the sky. The Observed Lunar Distance is calculated by spherical trigonometry from the Observed Positions as they appear on the sphere of the sky. In the sky sphere, the zenith point is analogous to the pole of the more familiar navigational triangle on the earth, and the horizon is analogous to the equator.

FIGURE 2

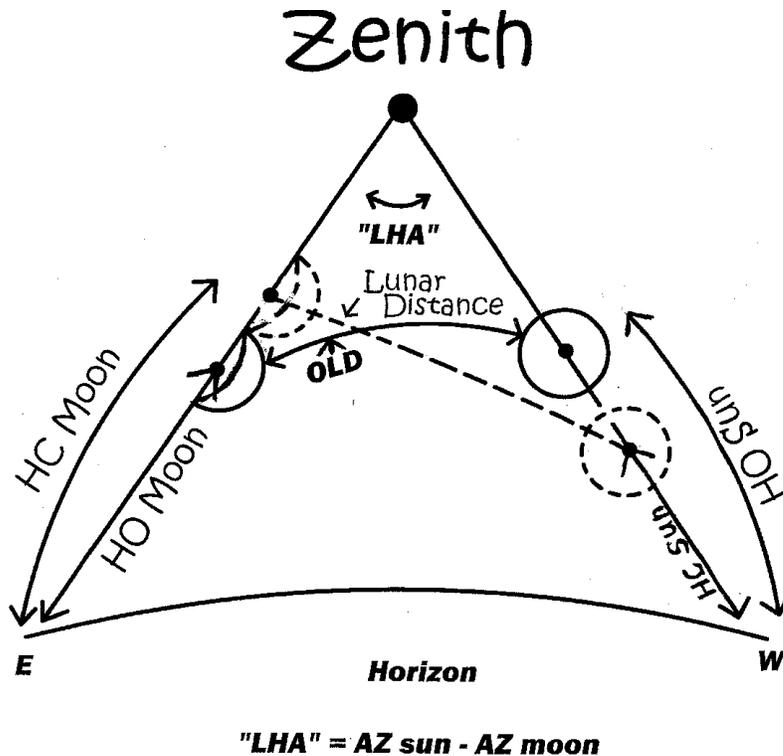


Figure two illustrates the Lunar Distance and Observed Lunar Distance between the sun and moon as seen from an observer with a sextant on land or in a boat. The dotted figures represent the geocentric positions of the sun and moon, the "HC's" as calculated from the Nautical Almanac. The solid figures represent the observed positions of the sun and moon in the sky, taking in account the effects of refraction and parallax. The altitude HC of the sun is increased by refraction to the observed altitude HO. The altitude HC of the moon is likewise increased by refraction, but decreased by the more significant effects of parallax, so that HO of the moon is less than HCmoon.

The first task is to calculate the HC and azimuth of each body for the date, assumed time and assumed position from data in the Nautical Almanac, using the basic formulas of spherical trigonometry as we did in the previous section. The calculated altitudes (HC) of the sun and moon are then increased by refraction from the atmosphere and decreased by parallax to yield the observed altitudes, HO. (Note that this is the opposite of how we normally work up a sight, where we subtract the refraction correction and add the parallax correction to our observed value HO, to get HC.) The azimuths of the sun and moon are unchanged by refraction and parallax. It is from these observed altitudes, and the difference in azimuths between the sun and moon, that the Observed Lunar Distance is calculated. These calculations cannot be done with any sort of inspection tables, as we mentioned.

Example: Calculate the OLD for May 7, 2000 at 23:38:00 GMT from Lat 47° 40.5' N, Lon 125° 53.9' W

We calculated the HC and azimuth AZ of the sun at this time and location in the example in Section 5. Similarly for the moon we get GHA = 119° 52.2', Dec = 21° 34.4' from the Almanac. LHA = GHA – Longitude = 119 52.2 - 125 53.9 = - 6°1.7'. From those values we calculate as in the previous section: HC = 63° 27 3', AZ = 167° 22 44'.

We now calculate the HO's, the observed values, for the sun and moon by correcting HC for refraction and parallax. We ignore parallax for the sun, as its impact is negligible here. The refraction corrections are taken from the table for stars and planets in the Nautical Almanac. The horizontal parallax HP of the moon is given hourly on the daily pages, and is 60.1'. The parallax correction is equal to  $HP\cos(HC)$  and is subtracted from HC. Parallax correction is  $60.1'\cos(63^\circ 27.1') = 26.9'$ .

HCsun =	39° 27.9'	HCmoon =	63° 27.1'
+ Refraction	+1.2'	-Parallax	-26.9'
HOsun =	39° 29.1'	+Refraction	0.5'
		HOmoon =	63° 0.7'

We can now calculate the Observed Lunar Distance between the centers of the sun and moon, applying the analogue of formula (1) on the spherical triangle of the sun, moon, and zenith in the sphere of the sky (see figure 2) remembering that  $\sin(90-\theta) = \cos(\theta)$ :

$$\text{Cos(OLD)} = \sin(\text{HOsun})\sin(\text{HOmoon}) + \cos(\text{HOsun})\cos(\text{HOmoon})\cos(\text{AZsun}-\text{AZmoon})$$

Plugging the numbers into the formula above we get:

$$\text{Cos(OLD)} = \sin(39^\circ 29.1')\sin(63^\circ 0.7') + \cos(39^\circ 29.1')\cos(63^\circ 0.7')\cos(250.237^\circ - 167.379^\circ)$$

$$\text{Cos(OLD)} = .61014$$

$$\text{OLD} = \cos^{-1}(.61017) = 52.397^\circ = 52^\circ 23.9'$$

But this isn't quite the OLD we measure with the sextant. This is the calculated OLD between the apparent centers of the sun and the moon. To get the actual measured OLD we need to subtract the semi-diameters of the sun and moon from this value. The semi-diameter of the sun is taken from the bottom of the Almanac page, 15.9' for this date. The semi-diameter SD of the moon is best calculated from the horizontal parallax HP given each hour in the Almanac by the formula  $SD = .2724HP$ . In this example  $SD = .2724 \times 60.1' = 16.4'$ . But we are still not done. This is the *geocentric* semi-diameter of the moon. The semi-diameter observed from the surface is generally increased by the fact that we are closer to the moon from the surface of the earth, about 4000 miles closer

when the moon is straight overhead. This *augmentation* to the semi-diameter is about  $0.3'\sin(HC)$ , or  $0.3'\sin(63^\circ) = 0.2'$ , approximately.

The OLD then is  $52^\circ 23.9' - 15.9' - 16.4' - 0.2' = 51^\circ 51.4'$

## 7. Determining GMT and Longitude at Sea from Observation of the Lunar Distance (OLD)

You need to make only two measurements to find longitude and time: the Observed Lunar Distance and the altitude of some body, all timed with the local clock that we are trying to calibrate for GMT. Any crude clock will do; Slocum used an old tin clock, which ran only sporadically.

The basic procedure for calculating GMT is an iterative method that starts off from an assumed time and position. The Latitude, of course, we know from the meridian passage of some body (or else we peek at the GPS, but only for the Latitude!). **Assume** a GMT for your local clock based on your best estimate of the time and find the corresponding assumed Longitude. The assumed Longitude comes from the intersection of the parallel of known Latitude with an LOP from an altitude sight using the assumed GMT. Or you can assume a Longitude based on your best Estimated Position and calculate the corresponding assumed time from the altitude of the observed body. The assumed time and the assumed Longitude are linked; if you increase the assumed time by four minutes, you increase the corresponding assumed Longitude by one degree toward the west. The procedure then is as follows:

- 1) From the assumed position at the assumed time, calculate the corresponding OLD. This is the inverse process of clearing the lunar distance. See Section six.
- 2) Compare the calculated OLD with the actual measured OLD. From the difference in these Lunar Distances, calculate the time correction to the assumed time. In the example below, I use the rate of 106 seconds of time/minute of arc of OLD difference ( $106 \text{ sec}/1'$ ), which is the actual rate of increase of geocentric Lunar Distance for the day and hour in the example. The rate of lunar distance change with time can vary from  $106\text{sec}/1'$  to  $136\text{sec}/1'$ . The exact value used for this correction doesn't matter much; any value around  $12.5^\circ/\text{day}$  ( $115\text{sec}/1'$ ) will work fine and converge to the correct answer. However, you must use a rate similar to the rate of change for the geocentric Lunar Distance; using the slope of the OLD observations regression line (in the earlier example in section 3,  $156\text{sec}/1'$ ) can converge very poorly.
- 3) Apply the time correction to the assumed time, and make the corresponding correction to the assumed Longitude. The assumed Longitude, of course, is increased to the west by  $1^\circ$  for every four minutes increase in assumed time, or  $1'$  for every four seconds of increased time. This correction produces a new assumed time, and a corresponding new assumed position.

- 4) Go back to step #1. When the difference between the calculated OLD and observed OLD is less than the measurement error (I generally use 0.1'), you are through, and have found both GMT and the Longitude.

We work out an example below: We will use the value of OLD of  $51^{\circ} 43' 36''$  actually taken at GMT 23:24:00 from Lat  $47^{\circ} 40.5' N$ , Lon  $122^{\circ} 23.9' W$  discussed in Section 3. However, let us imagine that the observations were made with a clock that was 14 minutes fast on GMT. The corresponding longitude would then be found to be  $3^{\circ} 30'$  to the west. Our **assumed time** would be 23:38:00, and the corresponding **Assumed Longitude** would be  $125^{\circ} 53.9'$ . The actual measured OLD is  $51^{\circ} 43' 36''$ . We used these values in our example in section 6 to calculate the OLD. As there are some rounding errors, we carry out the calculations to the nearest second:

Time	Longitude	Calculated OLD
1) 23:38:00	$125^{\circ} 53' 54''$	$51^{\circ} 51' 26''$

The lunar distance is increasing with time (moon waxing). The difference between calculated and measure OLD ( $\Delta$  OLD) is  $7' 50''$ , leading to a time correction ( $\Delta$  T) of  $-13' 52''$  and a corresponding longitude correction ( $\Delta$  L) of  $-3^{\circ} 27' 58''$ . Applying the time and longitude corrections and calculating the new OLD at the new time and longitude we get:

2) 23:24:08	$122^{\circ} 25' 56''$	$51^{\circ} 43' 29''$
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This time the  $\Delta$  OLD is  $-0' 7''$ , the corresponding  $\Delta$  T is  $12''$ , and the  $\Delta$  L is  $3'$ . Applying these corrections and calculating the new OLD, we get:

3) 23:24:20	$122^{\circ} 28' 56''$	$51^{\circ} 43' 36''$
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At this point the calculated OLD is the same as the measured OLD, and the calculation is complete. We find a GMT 20 secs fast, and a corresponding longitude  $5'$  to far west.

The method describe here converges very fast and accurately. It almost converged on the first iteration. The error here of 20 sec corresponds almost exactly to the observational error of 0.2'. If an assumed time with one hour error is used, with a corresponding longitude error of 15 degrees (up to 900 miles), this procedure still converges in two iterations to the same solution.

These calculations can be done with a Nautical Almanac and a simple scientific calculator, as demonstrated with these examples. I have also programmed a pocket Casio calculator with the very accurate Meeus algorithms for the sun and moon (*Astronomical Algorithms*, Jean Meeus, Willmann-Bell Inc. 1998), along with routines to calculate the OLD and to carry out the iteration procedure described above. Once I enter the date,

assumed time, assumed position, and observed OLD, the calculator returns the correct GMT and Longitude in 48 seconds. The StarPilot calculator from Starpath is the only commercially available programmed calculator I am aware of that can calculate lunars; it uses a similar approach to the one described here.

## **8. Finding an Actual Longitude at Sea**

In 1992 I had an opportunity to work a “lunar” during a sailing trip to Hawaii. It was my first ocean voyage, and I was looking forward to matching at least my celestial skills with those of Captain Slocum. The sky, however, wasn't cooperating. It was overcast for days, and I had to be content with grabbing quick sun lines whenever the solar disk resolved itself through the haze. Finally the sky cleared enough to reveal what I had been waiting for, the sun and crescent moon both visible "in distance" through an opening in the clouds.

I quickly broke out my sextant and roughly calculated the altitude of the moon, the lunar distance, and the altitude of the sun. The initial calculations would allow me to preset the sextant for these observations and save time "bringing down the sun," or what is even more awkward, bringing the image of the sun across the sky to touch the moon for measuring the lunar distance. I started my wind-up alarm clock with its sweep second hand to use as a time scale (you can still buy one at Payless, but it isn't made of tin) and prepared to take the sights. Slocum handled the whole process himself, but I enlisted the aid of an assistant to record clock times and sextant angles as I shouted them out. I made a series of measurements for averaging both the sun altitude and then the lunar distance. Fog obscured the horizon under the moon, but this iterative method of calculating lunars doesn't require a measurement of the moon altitude. I had practiced the lunar distance observation from the stability of my porch roof at home, but it is another thing to try it lying in the cockpit of a small boat and to brace yourself, squinting through the sextant telescope at the sky, in such a place that a sudden roll doesn't cause the mainsail to block out the image of the sun or the moon.

I assumed a GMT for the alarm clock based on an estimate of our approximate time. The sun was nearly due east when I took my sights, so the sun line at the assumed time gives a good corresponding assumed longitude. Calculating the HC's and azimuths of the sun and moon at the assumed time and position from data in the Nautical Almanac and clearing the lunar distance to adjust for refraction and parallax gives the calculated Observed Lunar Distance. Comparing the calculated OLD with the measured distance after two iterations gives Greenwich Mean Time and longitude of the sight. A classic Noon Sight for latitude advanced backwards provides a running fix. My position so calculated was less than seven miles from the truth.

Seven miles without a chronometer may not be up to modern electronic standards, but it sure would have given Columbus a start! In truth, a result this close must be somewhat fortuitous; my error in lunar distance of 0.3' was less than the specified accuracy of my inexpensive Astra sextant. Slocum claimed less than 5 miles error, and also freely admitted the role of luck. At least as far as my efforts go, Slocum's record still stands; it is the stuff of legends. In any case, I have never done better celestial work.

There are certainly better ways to get a position than the lunar method described here, or for that matter, quicker ways to cross an ocean than to be blown about in a sailboat. Staring at the dials of a GPS tells precisely where you are, but a carefully worked out lunar does more-it connects us with our past, with the navigators of the oceans and the mind who puzzled out the heavens and the earth. The GPS tells position, but a lunar fix reminds you also where you've come from. As Joshua Slocum wrote in 1899, "The work of the Lunarian, although seldom practiced in these days of chronometers, is beautifully edifying and there is nothing in the realm of navigation that lifts one's heart up more in adoration."