Distance to the Horizon

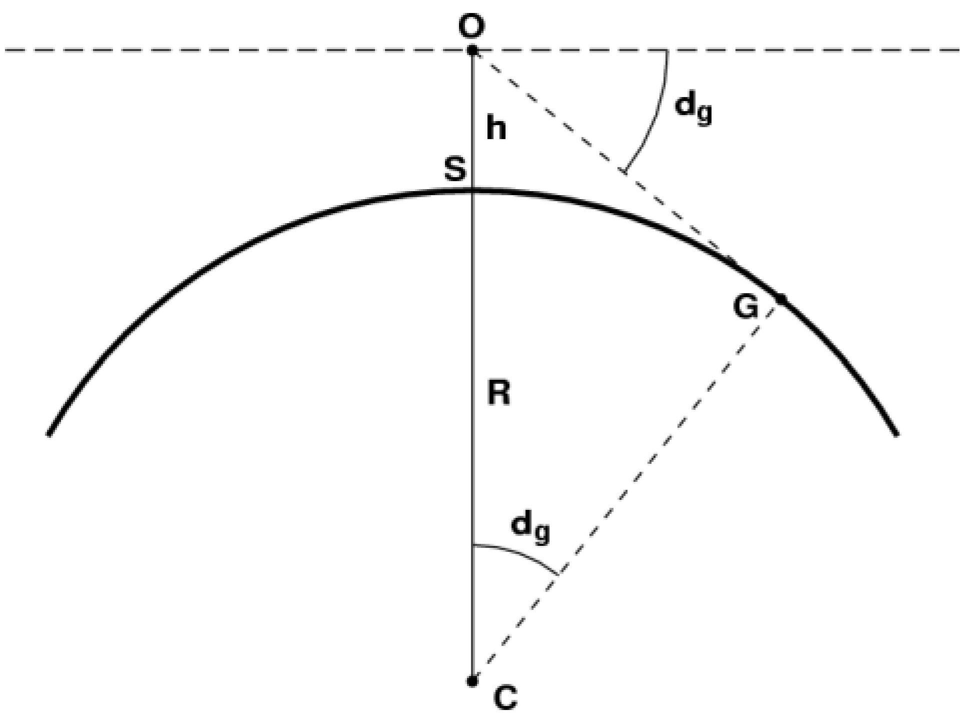
Introduction

People often ask how far you can see, or how far it is to the *apparent horizon*. These questions are somewhat different — obviously, you can see high mountains farther away than low hills or flat islands — but the answer is affected by *terrestrial refraction* in both cases.

Let's first consider these distances without refraction, and then add in the complications of varying refraction. Finally, we'll consider other atmospheric effects that play a part.

No refraction

Without refraction, the matter is very simple. Here's the diagram from my dip page:



It shows a vertical plane through the center of the Earth (at C) and the observer (at O). The radius of the Earth is R, and the observer's eye is a height h above the point S on the surface. (Of course, the height of the eye, and consequently the distance to the horizon, is greatly exaggerated in this diagram.) The observer's *astronomical horizon* is the dashed line through O, perpendicular to the Earth's radius OC. But the observer's *apparent horizon* is the dashed line OG, tangent to the surface of the Earth. The point G is the geometric horizon.

Elementary geometry tells us that, because the angle between the dashed lines at G is a right angle, the distance OG from the observer (O) to the horizon (G) is related to the radius R and the observer's height h by the Pythagorean Theorem:

$$(R + h)^2 = R^2 + OG^2$$

or

$$OG^2 = (R + h)^2 - R^2 .$$

But if we expand the term $(R + h)^2 = R^2 + 2 R h + h^2$, the R^2 terms cancel, and we find

$$OG = \text{sqrt} (2 R h + h^2) .$$

It's customary to use the fact that h << R at this point, so that we can neglect the second term. Then

$$OG \approx \text{sqrt} (2 R h)$$

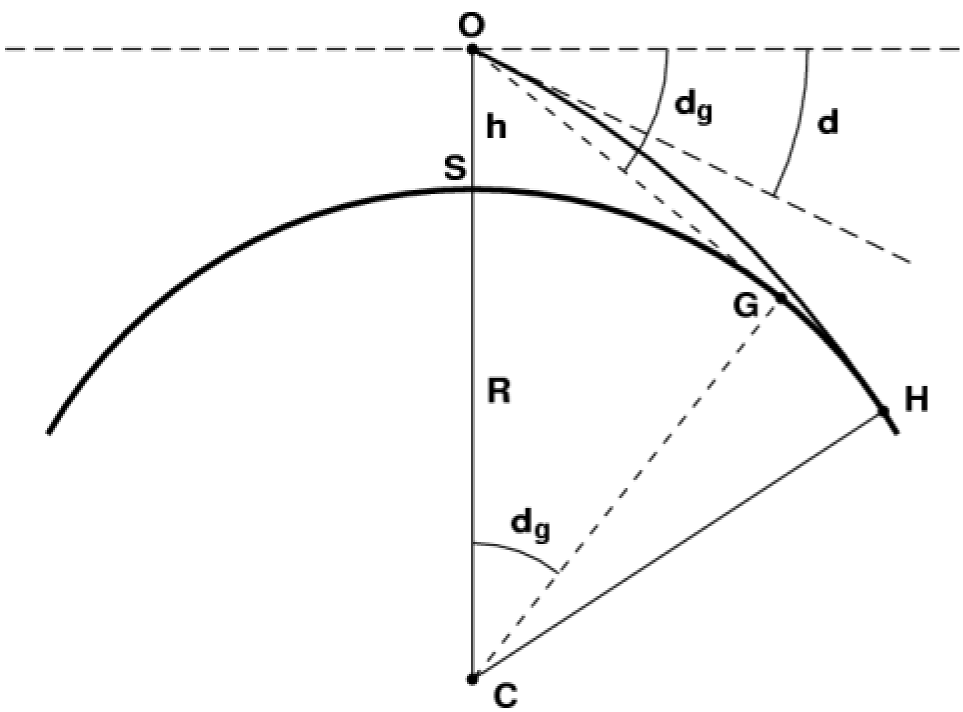
is the distance to the horizon, **neglecting refraction**.

Numerically, the radius of the Earth varies a little with latitude and direction; but a typical value is 6378 km (about 3963 miles). If h is in meters, that makes the distance to the geometric horizon 3.57 km times the square root of the height of the eye in meters (or about 1.23 miles times the square root of the eye height in feet).

Now let's allow for refraction.

Refraction, considered simply

Now, let's try to allow for refraction. Usually, the air is densest at the surface, so the rays of light are concave toward the surface; see the bending page for details.



The solid arc OH now represents the curved line of sight; H is the (refracted) apparent horizon. Notice that refraction lets us see a little farther, if the ray is concave toward the Earth, as shown here.

If we can assume a constant lapse rate in the air between the eye and the Earth's surface, and if the observer's height h is small compared to the 8-km height of the homogeneous atmosphere, then we can assume the curved ray is an arc of a circle. This assumption makes things easy, because the *relative* curvature of the ray and the Earth's surface is all that matters. In effect, we can use the previous result, but just use an effective radius of curvature for the Earth that is bigger than the real one.

This assumption is made so often that it's conventional in surveying and geodesy to use a “refraction constant” that's just the ratio of the two curvatures. A typical value of the ratio is about 1/7; that is, the ray curves about 1/7 as much as the Earth does (or, equivalently, the radius of curvature of the ray is about 7 times that of the Earth's surface).

Using this “typical” value means we should just use the formula given above, but use a value R′ instead of R for the effective radius of the Earth, where

$$1/R' = 1/R - 1/(7R) = 6/(7R) ,$$

so that

$$R' = R \times 7/6 .$$

That would make R′ about 7440 km, so that the distance to the horizon in kilometers is about 3.86 km times the square root of the height in meters (or about 1.32 miles times the square root of the height in feet).

In fact, this latter result (for English units) is nicely summed up by the **old rule** that 7 times the height in feet is 4 times the square of the distance to the horizon in miles; i.e.,

$$\text{distance to horizon (miles)} = \text{sqrt} [7 \times h \text{ (feet)} / 4].$$

NIMA has an on-line calculator that offers both English and metric units. Unfortunately, they put out far too many decimal places, giving a misleading impression of precision.

These approximations are all very handy; but how realistic are they?

Variable gradients

Unfortunately, the refraction varies considerably from day to day, and from one place to another. It is particularly variable over water: because of the high heat capacity of water, the air is nearly always at a different temperature from that of the water, so there is a thermal boundary layer, in which the temperature gradient is far from uniform.

Worse yet, these temperature contrasts are particularly marked near shore, where the large diurnal temperature swings over the land can produce really large thermal effects over the water, if there is an offshore breeze. This is particularly bad news for anyone standing on the shore and wondering how far out to sea a ship or island might be visible.

It gets worse. While the dip of the horizon depends only on an average temperature gradient, and so can be found from just the temperatures at the sea surface and at the eye, the distance to the horizon depends on the reciprocal of the mean reciprocal of the temperature gradient. But the structure of thermal boundary layers guarantees that there will be large variations in the gradient, even in height intervals of a few meters. This means that on two different days with the same temperatures at the eye and the water surface (and, consequently, the same dip), the distance to the horizon can be very different.

In conditions that produce superior mirages, there are inversion layers in which the ray curvature exceeds that of the Earth. Then, in principle, you can see infinitely far — there really is no horizon.

Of course, we all know that visibility is limited by the clarity or haziness of the air. And the *duct* that (in principle) might allow you to see around the whole Earth doesn't really extend that far; it typically exists for some limited region, perhaps a few tens or a few hundreds of kilometers.

So the nice-looking formulae for calculating “the distance to the horizon” are really only rough approximations to the truth. You can consider them accurate to a few per cent, most of the time. But, occasionally, they will be wildly off, particularly if mirages are visible. Then it's common to see much farther than usual — a condition known as *looming*.

How far can you see?

Still, even with those caveats, it's of interest to consider how far the eye can see under different conditions. Usually the visibility is limited by scattered light in the lower atmosphere; see

> Craig F. Bohren and Alistair B. Fraser
> “At what altitude does the horizon cease to be visible?”
> *American Journal of Physics* **54**, Issue 3 (March, 1986) p. 222

for details. Even under extremely clear conditions, it's unusual to see more than a couple of hundred kilometers.

However, there is one situation in which objects can be made out at great distances: when they are silhouetted against a bright background, such as the setting Sun, or (just after sunset) a bright twilit sky.

Here in San Diego, we rarely can see San Clemente Island, about 125 km offshore. The top of the island should just be visible above our horizon with normal refraction, but it's concealed by “airlight” during the day. Even in the clear air of a “Santa Ana,” which causes looming and raises more of the island above the apparent horizon, it's often hard to make out.

But just after sunset, the island is often visible, if you know where to look. The air between you and the island is only dimly illuminated after sunset, but the sky behind the island — i.e., the air beyond the horizon that is still in direct sunlight — is still fairly bright. Then the silhouette of the island is striking, even if it had been invisible a few minutes before sunset.

The Sun itself **can be seen** through a long duct when it is several degrees below the astronomical horizon; however, its image is then so **distorted** that any intervening terrestrial object (such as an island, a mountain, or even a cloud) would probably also be so distorted that its silhouette against the Sun would be unrecognizable. Some extreme claims can surely be discounted, such as **Jessen's 1914** illusion. (Jessen claimed to have seen a mountain nearly 900 km away, but he certainly did not; **Korzenewsky (1923)**, who refers to this report in a footnote, somehow inflated that to 1177 km.)

What's the record for visibility without help from the silhouetting effect? I think that might belong to the report of the expedition led by **Korzenewsky (1923)**, who reported seeing snow-capped peaks of a mountain range 750 km away. Conditions were perfect: the lower atmosphere was in shadow at sunset; the peaks were quite high (4650 meters, or over 15,000 feet); they were covered with white snow, increasing their visibility; and there must also have been considerable looming to bring these distant features above the observers' horizon. As the observation was made on June 1, near the peak of superior-mirage season, the looming is not improbable, though the amount required is hard to believe. The observers themselves were in the deserts of Turkestan [now southeastern Kazakhstan] at a height of nearly a kilometer, where the dryness of the air favored extreme clarity, and looking across a broad, sandy depression. And, of course, much of the air path was in thinner air well above ground level, because of the mountains' height.

For less extreme, but very reliable, observations, consider some listed by Commander C. L. Garner of the Coast and Geodetic Survey in **1933**. He says that instrumental measurements were made in both directions “between Mt. Shasta and Mt. St. Helena in California, a distance of 192 miles.” [That's 309 km.] Apparently this was done in normal conditions, with no looming; heliotropes having 12-inch [30-cm] mirrors were used. He also credits the 1911 sighting of the Fairweather Mountains in Alaska from the ship *Explorer* from the Gulf of Alaska, 330 miles [531 km] away.

If you'd like to explore the consequences of various (constant) values for the lapse rate, I have a JavaScript calculator here. It uses the simplistic circular-ray approximation, so take its calculations with a grain of salt.

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