Notes on Bearing Fix Accuracy

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An example of a 3 LOP fix, from 3 compass bearings to 3 light houses from a vessel dead in the water in calm air. The compass was a hocky puck (Mini Moran 2000) which can typically take bearings in good conditions to $\pm 1^{\circ}$ or so, providing there is no unknown interference from, for example, eye glasses or maybe a watch. If such errors are present, we cannot offhand conclude they are the same for each bearing as they would create what is effectively deviation, which depends on heading. The bearings were taken by an experienced navigator, so we might assume these were nor present.

The measurement illustrates several points we have discussed. The key point here is the varying distances to each mark. They are 1.5 nmi, 3.5 nmi, and 8.5 nmi, as shown by the added red tick marks.



If we assume there is $a \pm 1^{\circ}$ uncertainty in each bearing, the error at the vessel location will depend on the distance from it. We can use our simple 6° rule to evaluate this, namely 6° compass error (or set angle) causes a cross track error of 10% of distance run. So the uncertainties are $\pm 1.5 \times 0.1/6$, $3.5 \times 0.1/6$, and $8.5 \times 0.1/6 = 0.03$ nmi, 0.06 nmi, and 0.14 nmi, which equals 60 yd, 120 yd, and 280 yds.

Now let's look at a larger scale view.



In this picture the error bars are plotted, showing that the sight from 8.5 nmi off is much larger than those from 1.5 nmi off, which illustrates the rule, take bearings to close targets when possible. Also note that the bearings to the 8.5 mi and 3.5 mi targets were not that much different in bearings, though opposite directions, which means the cut angle is very narrow—almost at the limit here, but this one is usable. Notice that if you just had these two, the uncertainty would be very large (overlap of pink and blue). What saves the fix is the bearing to the close target (uncertainty in yellow). The true GPS position of the boat was the boat icon, circled in red here. notice that we barely touch it with our uncertainties, so we were likely optimistic with $\pm 1^{\circ}$. Twice that might have been closer.

If we had to guess our position from this fix, we would <u>not</u> want to use the center of the "cocked hat" of LOPs, because the uncertainties from each line are so different. We would better chose the region where the uncertainties overlap, marked here with a red triangle. Thus it is centered on the most accurate line, half way between the other two.

The details we need to add here are a statistical analysis to show that you are not necessarily inside the triangle even when the uncertainties are about the same, yet still random, and then to show that we are still best off with 3 bearings about 120° apart because that is the unique case that will cancel a constant error that might apply to each one. A constant error that affects each sight is more likely in celestial nav sights than with compass bearings, but it is not out of the question. A lubber's line error is a constant error to each one, as is a personal bias to read each of the angles too high, for example.

These measurements are from Andrés Ruiz, who is the author of Navigational Algorithms, at http://sites.google. com/site/navigationalalgorithms. He has made many very nice contributions to navigators, worldwide. He posted this data in the NavL discussion group on Dec 16, 2010. I have just added the discussion and analysis. db



Three bearings taken 120° apart has the great virtue that a constant error that applies to each sight will cancel out. The error makes the triangle larger but the center of the triangle is still an accurate fix. In the top picture, the black lines are perfect sights. The red lines are each wrong by $+6^{\circ}$.

In the bottom picture we have the same errors applied to three sights taken 60° apart. This leads to a triangle (cocked hat) of LOPs that has exactly the same shape as the ones taken 120° apart, but now notice that the proper fix is not located inside the triangle.

Thus the rules for best fixes are:

- (1) close targets rather than far
- (2) permanent rather than floating
- (3) three targets as near 120° apart as possible

(4) if you are moving, take the ones first that are changing bearing the least.